

# Chiral-odd transversity GPDs from a leading twist hard amplitude

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The chiral-odd transversity generalized parton distributions (GPDs) of the nucleon can be accessed experimentally through the exclusive photoproduction process  $\gamma + N \rightarrow \pi + p + N'$ , in the kinematics where the meson pair has a large invariant mass and the final nucleon has a small transverse momentum, provided the vector meson is produced in a transversally polarized state. We calculate perturbatively the scattering amplitude at leading order in  $\alpha_s$ . We build a simple model for the dominant transversity GPD  $H_T(x, \xi, t)$  based on the concept of double distribution. We estimate the unpolarized differential cross section for this process in the kinematics of the Jlab and COMPASS experiments. Counting rates show that the experiment looks feasible with the real photon beam characteristics expected at JLab@12 GeV, and with the quasi real photon beam in the COMPASS experiment.

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## 1. Chiral-odd GPDs and factorization

Transversity quark distributions in the nucleon remain among the most unknown leading twist hadronic observables, mainly due to their chiral-odd character which enforces their decoupling in most hard amplitudes. After the pioneering studies of Ref. [1], much work [2] has been devoted to the study of many channels but experimental difficulties have challenged the most promising ones.

On the other hand, tremendous progress has been recently witnessed on the QCD description of hard exclusive processes, in terms of generalized parton distributions (GPDs) describing the 3-dimensional content of hadrons. Access to the chiral-odd transversity GPDs [3], noted  $H_T$ ,  $E_T$ ,  $\tilde{H}_T$ ,  $\tilde{E}_T$ , has however turned out to be even more challenging [4] than the usual transversity distributions: one photon or one meson electroproduction leading twist amplitudes are insensitive to transversity GPDs. The strategy which we follow here, as initiated in Ref. [5], is to study the leading twist contribution to processes where more mesons are present in the final state<sup>1</sup>; the hard scale which allows to probe the short distance structure of the nucleon is  $s = M_{\pi\rho}^2 \sim |t'|$  in the fixed angle regime. In the example developed previously [5], the process under study was the high energy photo (or electro) diffractive production of two vector mesons, the hard probe being the virtual "Pomeron" exchange (and the hard scale was the virtuality of this pomeron), in analogy with the virtual photon exchange occurring in the deep electroproduction of a meson.

We study here [7] a process involving a transversely polarized  $\rho$  meson in a 3-body final state:

$$\gamma + N \rightarrow \pi + \rho_T + N'. \quad (1.1)$$

It is a priori sensitive to chiral-odd GPDs due to the chiral-odd character of the leading twist distribution amplitude (DA) of  $\rho_T$ . The estimated rate depends of course much on the magnitude of the chiral-odd GPDs. Not much is known about them, but model calculations have been developed in Refs. [5, 8–10] and a few moments have been computed on the lattice [11]. To factorize the amplitude of this process we use the now classical proof of the factorization of exclusive scattering at fixed angle and large energy [12]. The amplitude for the process  $\gamma + \pi \rightarrow \pi + \rho$  is written as the convolution of mesonic DAs and a hard scattering subprocess amplitude  $\gamma + (q + \bar{q}) \rightarrow (q + \bar{q}) + (q + \bar{q})$  with the meson states replaced by collinear quark-antiquark pairs. This is described in Fig.1a. The absence of any pinch singularities (which is the weak point of the proof for the generic case  $A + B \rightarrow C + D$ ) has been proven in Ref. [13] for the case of interest here. We then extract from the factorization procedure of the deeply virtual Compton scattering amplitude near the forward region the right to replace in Fig.1a the lower left meson DA by a  $N \rightarrow N'$  GPD, and thus get Fig.1b. The needed skewness parameter  $\xi$  is written in terms of the meson pair squared invariant mass  $M_{\pi\rho}^2$  as

$$\xi = \frac{\tau}{2 - \tau}, \quad \tau = \frac{M_{\pi\rho}^2}{S_{\gamma N} - M^2}. \quad (1.2)$$

Indeed the same collinear factorization property bases the validity of the leading twist approximation which either replaces the meson wave function by its DA or the  $N \rightarrow N'$  transition by nucleon GPDs. A slight difference is that light cone fractions  $(z, 1 - z)$  leaving the DA are positive, while the corresponding fractions  $(x + \xi, \xi - x)$  may be positive or negative in the case of the GPD. A Born order calculation show that this difference does not ruin the factorization property. In order

<sup>1</sup>A similar strategy has also been advocated recently in Ref. [6] for chiral-even GPDs.

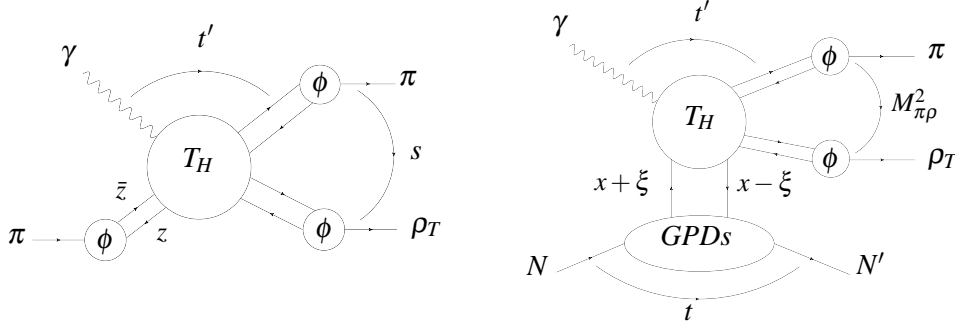


Figure 1: a) (left) Factorization of the amplitude for  $\gamma + \pi \rightarrow \pi + \rho$  at large  $s$  and fixed angle; b) (right) replacing one DA by a GPD leads to the factorization of the amplitude for  $\gamma + N \rightarrow \pi + \rho + N'$  at large  $M_{\pi\rho}^2$ .

for the factorization of a partonic amplitude to be valid, and the leading twist calculation to be sufficient, one should avoid the dangerous kinematical regions where a small momentum transfer is exchanged in the upper blob, namely small  $t' = (p_\pi - p_\gamma)^2$  or small  $u' = (p_\rho - p_\gamma)^2$ , and the resonance regions for each of the invariant squared masses  $(p_\pi + p_{N'})^2, (p_\rho + p_{N'})^2, (p_\pi + p_\rho)^2$ .

## 2. The scattering amplitude

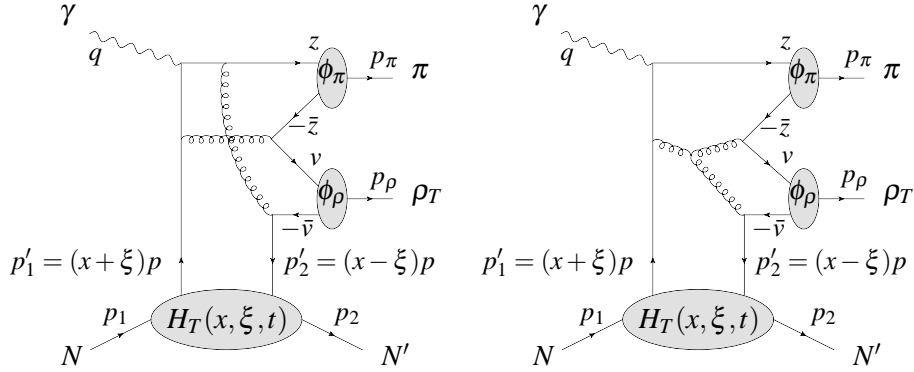


Figure 2: Two representative diagrams without (left) and with (right) three gluon coupling.

The scattering amplitude of the process (1.1) is written in the factorized form :

$$\mathcal{A}(t, M_{\pi\rho}^2, p_T) = \int_{-1}^1 dx \int_0^1 dv \int_0^1 dz T^q(x, v, z) H_T^q(x, \xi, t) \Phi_\pi(z) \Phi_\perp(v), \quad (2.1)$$

where  $T^q$  is the hard part of the amplitude and the transversity GPD of a parton  $q$  in the nucleon target which dominates at small momentum transfer is defined by [3]

$$\langle N'(p_2), \lambda' | \bar{q} \left( -\frac{y}{2} \right) \sigma^{+j} \gamma^5 q \left( \frac{y}{2} \right) | N(p_1), \lambda \rangle = \bar{u}(p', \lambda') \sigma^{+j} \gamma^5 u(p, \lambda) \int_{-1}^1 dx e^{-\frac{i}{2}x(p_1^+ + p_2^+)y^-} H_T^q,$$

where  $\lambda$  and  $\lambda'$  are the light-cone helicities of the nucleon  $N$  and  $N'$ . The chiral-odd DA for the transversely polarized meson vector  $\rho_T$ , is defined, in leading twist 2, by the matrix element

$$\langle 0 | \bar{u}(0) \sigma^{\mu\nu} u(x) | \rho_T^0(p, \epsilon_\pm) \rangle = \frac{i}{\sqrt{2}} (\epsilon_\pm^\mu(p) p^\nu - \epsilon_\pm^\nu(p) p^\mu) f_\rho^\perp \int_0^1 du e^{-iup \cdot x} \phi_\perp(u),$$

where  $\epsilon_\pm^\mu(p)$  is the  $\rho$ -meson transverse polarization and with  $f_\rho^\perp = 160$  MeV. Two classes of

Feynman diagrams (see Fig.2), without and with a 3-gluon vertex, describe this process. The scattering amplitude gets both a real and an imaginary parts. Integrations over  $\nu$  and  $z$  have been done analytically whereas numerical methods are used for the integration over  $x$ .

### 3. Results

Various observables can be calculated with this amplitude. We stress that even the unpolarized differential cross-section  $\frac{d\sigma}{dt du' dM_{\pi p}^2}$  is sensitive to the transversity GPD. To estimate the rates, we modelize the dominant transversity GPD  $H_T^q(x, \xi, t)$  ( $q = u, d$ ) in terms of double distributions

$$H_T^q(x, \xi, t = 0) = \int_{\Omega} d\beta d\alpha \delta(\beta + \xi\alpha - x) f_T^q(\beta, \alpha, t = 0), \quad (3.1)$$

where  $f_T^q$  is the quark transversity double distribution written as

$$f_T^q(\beta, \alpha, t = 0) = \Pi(\beta, \alpha) \delta q(\beta) \Theta(\beta) - \Pi(-\beta, \alpha) \delta \bar{q}(-\beta) \Theta(-\beta), \quad (3.2)$$

where  $\Pi(\beta, \alpha) = \frac{3}{4} \frac{(1-\beta)^2 - \alpha^2}{(1-\beta)^3}$  is a profile function and  $\delta q, \delta \bar{q}$  are the quark and antiquark transversity parton distribution functions of Ref. [14]. The  $t$ -dependence of these chiral-odd GPDs - and its Fourier transform in terms of the transverse localization of quarks in the proton [15] - is very interesting but completely unknown. We describe it in a simplistic way using a dipole form factor:

$$H_T^q(x, \xi, t) = H_T^q(x, \xi, t = 0) \times \frac{C^2}{(t - C)^2} \quad (C = .71 \text{ GeV}^2). \quad (3.3)$$

In Fig. 3, we show the  $M_{\pi p}^2$  dependence of the differential cross section  $d\sigma/dM_{\pi p}^2$ . Let us first

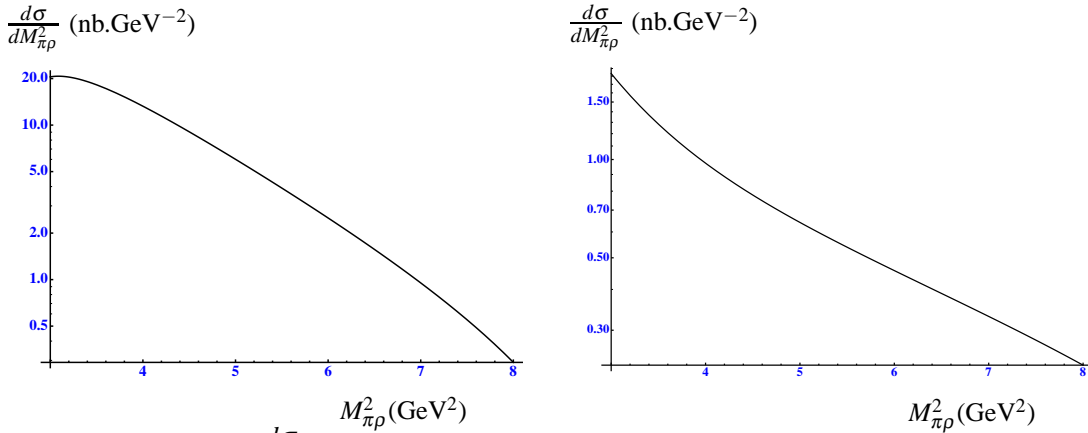


Figure 3:  $\frac{d\sigma}{dM_{\pi p}^2}$  (nb.GeV $^{-2}$ ) at  $S_{\gamma N} = 20 \text{ GeV}^2$  (left) and  $S_{\gamma N} = 100 \text{ GeV}^2$  (right).

discuss the specific case of muoproduction with the COMPASS experiment at CERN. Integrating differential cross sections on  $t, u'$  and  $M_{\pi p}^2$ , with justified cuts on  $t'$  and  $u'$  and  $M_{\pi p}^2 > 3 \text{ GeV}^2$ , leads to an estimate of the cross sections for the photoproduction of a  $\pi^+ \rho_T^0$  pair at high energies such as

$$\sigma_{\gamma N \rightarrow \pi^+ \rho_T^0 N'}(S_{\gamma N} = 100 \text{ GeV}^2) \simeq 3 \text{ nb}. \quad (3.4)$$

The virtuality  $Q^2$  of the exchanged photon plays no crucial role in our process, and the virtual photoproduction cross section is almost  $Q^2$ -independent if we choose to select events in a sufficiently narrow  $Q^2$ -window ( $.02 < Q^2 < 1 \text{ GeV}^2$ ), which is legitimate since the effective photon flux is

strongly peaked at very low values of  $Q^2$ . This yields a rate sufficient to get an estimate of the transversity GPDs in the region of small  $\xi$  ( $\sim 0.01$ ). In the lower energy domain, which will be studied in details at JLab, and with the same cuts as above, estimates of the cross sections are:

$$\sigma_{\gamma N \rightarrow \pi^+ \rho_T^0 N'}(S_{\gamma N} = 10 \text{ GeV}^2) \simeq 15 \text{ nb} \quad \sigma_{\gamma N \rightarrow \pi^+ \rho_T^0 N'}(S_{\gamma N} = 20 \text{ GeV}^2) \simeq 33 \text{ nb}. \quad (3.5)$$

Thanks to the high electron beam luminosity expected at JLab, a detailed analysis should be possible, both in Hall B and Hall D.

In conclusion, we expect the process discussed here to be observable in two quite different energy ranges, which should give complementary information on the chiral-odd transversity GPDs: the large  $\xi$  region may be scrutinized at JLab and the smaller  $\xi$  region may be studied at COMPASS. Let us stress that our study is built on known leading twist factorization theorems, contrarily to other attempts to access transversity GPDs [16].

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